Lineability of Nowhere Monotone Measures

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We say that a finite signed Radon measure on a d-dimensional real space $\mathbb{R}^d$ is *nowhere monotone* if

$$\mu^+(G) > 0,$$
$$\mu^-(G) < 0,$$

for every non-empty open set $G \subset \mathbb{R}^d$. We will show that the set of all such measures is residual in the space of signed Radon measures on $\mathbb{R}^d$ and thus "'big"' in the sense of set category.

We will follow up on this result by showing that the set of nowhere monotone signed Radon measures on a d-dimensional real space $\mathbb{R}^d$ is *lineable*. More specifically, we prove that there exists a vector space of dimension $\mathfrak{c}$ (the cardinality of the continuum) of signed Radon measures on $\mathbb{R}^d$ every non-zero element of which is a nowhere monotone measure that is almost everywhere differentiable with respect to the d-dimensional Lebesgue measure.

Using this result we further show that there exists a subspace of these measures that is dense in the space of bounded signed Radon measures on $\mathbb{R}^d$ that are almost everywhere differentiable with respect to the d-dimensional Lebesgue measure and that the cardinality of the basis of this subspace is the maximal possible.

All of the results mentioned above are recent and form the basis of the author's article "'*Lineability of Nowhere Monotone Measures*"' which has been recently admitted for publication in "'*Bulletin of the Belgian Mathematical Society Simon Stevin*"'.