# SWAM Prague, Czech Republic November 17-19, 2022

# Conference Booklet



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CHARLES UNIVERSITY

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# Schedule

IT: Invited Talk, CT: Contributed Talk, SI: Short Introduction.

# Thursday, 17<sup>th</sup> November

K1 lecture room, Campus Karlín

8:15-8:50	Registration				
8:50-9:00		Workshop opening			
			Chair: Tomislav Rožić		
9:00-10:30	IT	<b>Rolf Poulsen</b> Copenhagen, Denmark	Tools for Modern Computational Finance		
10:30-11:00	Coffee break				
			Chair: Jakob Zech		
11:00–11:45	IT	<b>Jens Saak</b> Magdeburg, Germany	A Journey to Digital Twins of Machine Tools		
11:45-12:30	IT	<b>Melina Freitag</b> Potsdam, Germany	From Models to Data and Back - An Introduction to Data Assimilation Algorithms		
12:30-13:45		Lunch break			
			Chair: Jens Saak		
13:45-14:05	СТ	Jonas Schulze Magdeburg, Germany	Diagonally Addressed Matrices		
14:05–14:25	СТ	<b>Petr Vacek</b> Prague, Czechia	Stopping Criteria for Coarsest-grid Solver in Multigrid V-cycle Method		
14:25–14:45	СТ	<b>Mohamed Amine Hamadi</b> Dunkirk, France	A Data-driven Krylov Model Order Reduction for Large-Scale Dynamical Systems		
14:45-15:00	СТ	<b>Mariia Vlasiuk</b> Lviv, Ukraine	On the Integral Equations Approach for Solving Nonlinear Inverse Elastostatics Problem		
15:00–15:15	SI	Student Introduction	Students will introduce themselves in at most 4 minutes each.		
15:15-15:45		Coffee break			
15:45-17:15	SI	Student Introduction	Students will introduce themselves in at most 4 minutes each.		
19:00		Confer	ence dinner		

# Friday, 18<sup>th</sup> November

S3 lecture room, Malá Strana Campus

			Chair: Melina Freitag	
0.00 0.45	ТТ	Roman Neruda     Prague, Czechia     Cristóbal Bertoglio     Groningen, Netherlands     Col     Jan Papež     Prague, Czechia     Martin Stoll     Chemnitz, Germany     Jakob Zech     Heidelberg, Germany     Col     Prague, Czechia     Oldřich Semerák     Prague, Czechia	Adversarial Examples: How Safe are	
9.00-9.43	11	Prague, Czechia	Machine Learning Models	
9:45-10:30	IT	Cristóbal Bertoglio	Inverse Problems in Blood Flow Modeling	
		Groningen, Netherlands		
10:30-11:00		Coffee break		
			Chair: Roman Neruda	
11:00-11:45	IT	Jan Papež	Can a Computed Pagult be Palied on?	
11.00-11.45	11	Prague, Czechia	Can a Computed Result be Relied on?	
11.45 12.20	IT Cristóbal Berto Groningen, Nether   00 IT   45 IT   30 IT   45 IT   30 IT   30 IT   IT Jan Papež Prague, Czech Martin Stoll Chemnitz, Germ   45 IT   15 IT   Jakob Zech Heidelberg, Germ   30 IT   Petr Minaříl Prague, Czech	Martin Stoll	From PDEs to Data Science: An	
11.45-12.50		Chemnitz, Germany	Adventure with the Graph Laplacian	
12:30–13:45		Lunch break		
			Chair: Martin Stoll	
13:45–15:15	IT	Jakob Zech	Large Scale Bayesian Inversion:	
		Heidelberg, Germany	Sparse-Grid and Transport Methods	
15:15-15:45		Coffee break		
			Chair: Friedemann Kemm	
15:45-16:30	IT	Petr Minařík	Realtime Interactive Volumetric Water	
		Prague, Czechia	Simulation	
16:30 17:15	IT	Oldřich Semerák	On the Marmoreally Beautiful and yet	
10.30-17.13		Prague, Czechia	Unlovely Sides of Einstein Equations	
17:15-17:30	Coffee break			
17:30			City tour	

# Saturday, 19<sup>th</sup> November

K1 lecture room, Campus Karlín

			Chair: Martin Sýkora	
9:00-10:30	IT	Friedemann Kemm Cottbus, Germany	How to Prepare a Scientific Presentation	
10:30-11:00		Coffee break		
			Chair: Jana Brunátová, Purusharth Saxena	
11:00-11:20	СТ	<b>Azza Gaysin</b> Prague, Czechia	Proof Complexity of CSP	
11.20 11.40	СТ	Josephine Westermann	Transport Based Sampling Using	
11:20–11:40	CI	Heidelberg, Germany	Polynomial Density Surrogates	
11.40 12.00	СТ	Liya Gaynutdinova	Micromechanical Parameter Identification	
11.40-12.00	CI	Prague, Czechia	Micromechanical Parameter Identification Using Bayes Method	
12:00-12:20	СТ	Muhammad Sami Siddiqui Karachi, Pakistan	Euler Elastica Model & Its Treatment	
Panasun Manorost Positive Preserving F	Positive Preserving Finite Volume Scheme			
12.20-12.40	CI	Heidelberg, Germany	Coffee break Chair: Jana Brunátová, Purusharth Saxena Proof Complexity of CSP Transport Based Sampling Using Polynomial Density Surrogates Micromechanical Parameter Identification Using Bayes Method Euler Elastica Model & Its Treatment Positive Preserving Finite Volume Schem for Surface/Subsurface Flow Simulation Time-Varying Semidefinite Programming: Path Following a Burer–Monteiro Factorization osing remarks	
12:40 12:00	СТ	Antonio Bellon	Time-Varying Semidefinite Programming:	
12.40-13.00	CI	Prague, Czechia	Path Following a Burer-Monteiro Factorization	
13:00-13:10	Closing remarks			

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# Presentation

The Student Workshop on Applied Mathematics (SWAM) is organized by SIAM Student Chapters Prague and Heidelberg, and the University of Copenhagen Association of Ph.D. Students. The workshop is primarily funded by the 4EU+ Alliance. Support from Charles University, Heidelberg University, and the University of Copenhagen is also acknowledged.

The purpose of this workshop is to bring graduate students studying applied mathematics in natural sciences, engineering, and mathematics together and create an environment to share ideas from different perspectives and to enable easier communication with researchers for possible interdisciplinary international collaboration.

# **Organizing Committee**

JANA BRUNÁTOVÁ Charles University, Czechia EDA OKTAY Charles University, Czechia PURUSHART SAXENA Heidelberg University, Germany MARTIN SÝKORA Charles University, Czechia TOMISLAV ROŽIĆ University of Copenhagen, Denmark

# **Abstracts of Invited Talks**

# **Tools for Modern Computational Finance**

#### **Rolf Poulsen<sup>1</sup>**

<sup>1</sup> University of Copenhagen, Copenhagen, Denmark

Under very general conditions – but for surprisingly subtle reasons – a key part finding a sensible price for any financial contract is the calculation of expected values. After briefly reviewing the arbitrage-free pricing of a call option in the Black-Scholes model, I describe some tools used in modern computational finance. More specifically, but deliberately teasingly, this will involve some of the following concepts and names:

- 1. Scripting languages and smart contracts
- 2. Fourier inversion and the return of complex analysis
- 3. Sobol, Longstaff-Schwartz, Craig-Sneyd
- 4. Automatic adjoint differentiation
- 5. Differential machine learning

#### References

[1] Antoine Savine et al. (2019-23), Modern Computational Finance, volumes 1-3, Wiley

# A Journey to Digital Twins of Machine Tools

#### Jens Saak<sup>1</sup>

<sup>1</sup>Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany

Industry 4.0 requires the design and control of complex models, which generally need to be fitted to measurement data and made suitable for real-time control applications; e.g., digital twins and digital shadows serve as computer-based representations of machines and processes during the entire product cycle. In this talk, we will evaluate strategies for modeling and model order reduction of machine tools, i.e. machines that are used to build (parts of) machines. Thermo-mechanical models for the machine tools feature a couple of challenges for both modeling and order reduction techniques:

- 1. To limit the effort, usually stationary elasticity is coupled with transient heat equations, rendering the full model a differential algebraic equation.
- 2. While the heat distribution and elastic deformation can be modeled by linear partial differential equations, some components like bearings and guide shoes feature nonlinearities that are crucial for correct representation of the machine behavior.
- 3. Modern machine tools feature up to 6 axes along which parts can be moved for optimal placement of the tool relative to the work-piece. On each of these axes, two parts of the machine are moving relative to each other making the entire model time-varying.

Over the recent decade, we have worked out methods to address all these challenges and created an automated workflow for practitioners using both proprietary and open-source tools to create and use the desired digital representation for each task.

## From Models to Data and Back - An Introduction to Data Assimilation Algorithms

#### Melina Freitag<sup>1</sup>

<sup>1</sup>University of Potsdam, Potsdam, Germany

Data assimilation is a method that combines observations (e.g. real world data) of a state of a system with model output for that system in order to improve the estimate of the state of the system. The model is usually represented by discretised time dependent partial differential equations. The data assimilation problem can be formulated as a large scale Bayesian inverse problem. Based on this interpretation we derive the most important variational and sequential data assimilation approaches, in particular three-dimensional and four-dimensional variational data assimilation (3D-Var and 4D-Var), and the Kalman filter. The final part reviews advances and challenges for data assimilation.

# **Adversarial Examples: How Safe are Machine Learning Models**

#### Roman Neruda<sup>1</sup>

<sup>1</sup>Czech Academy of Sciences, Prague, Czechia

Adversarial examples are special inputs to machine learning models intentionally designed to cause the model to make a mistake. They represent an important issue concerning security and generalisation capabilities of models deployed in practical applications. In the talk we will present several approaches of adversarial attacks for image classifiers, and sketch possible defense strategies.

# **Inverse Problems in Blood Flow Modeling**

#### Cristóbal Bertoglio<sup>1</sup>

<sup>1</sup>University of Groningen, Groningen, Netherlands

Mathematical and computational modeling of the cardiovascular system is increasingly providing noninvasive alternatives to traditional invasive clinical procedures. Moreover, it has the potential for generating additional diagnostic markers. In blood flow computations, the personalization of spatially distributed models is a key step which relies on the formulation and numerical solution of inverse problems using clinical data. In the last years, the development and application of inverse methods has rapidly expanded, see [1], most likely due to the increased availability of data in clinical centers and the growing interest of modelers and clinicians in collaborating. In this talk I will present some of the inverse problems in blood flows formulated in our group, in particular from data coming from Magnetic Resonance Imaging and including fluid-structure interaction models.

#### References

[1] D. Nolte, C. Bertoglio, Inverse Problems in Blood Flow Modeling: A review, *International Journal for Numerical Methods in Biomedical Engineering* **38(8)**, e3613 (2022), 10.1002/cnm.3613.

# Can a Computed Result Be Relied On?

#### Jan Papež<sup>1</sup>

<sup>1</sup>Czech Academy of Sciences, Prague, Czechia

Real-world applications often give rise to complex and challenging problems that can only be solved numerically. In order to reduce the complexity of the problem and allow us to get an approximation within the given time using available computational resources, one has to consider some simplifications, which typically introduce various errors. We should then ask how accurate is the computed approximation and whether it can be used for the original application. This can be answered by so-called a posteriori error estimates that are based on the properties of the problem to be solved and evaluated from the computed approximation. In the talk, we will present some error estimates for numerical solution of partial differential equations.

## From PDEs to Data Science: An Adventure with the Graph Laplacian

#### Martin Stoll<sup>1</sup>

<sup>1</sup>Chemnitz University of Technology, Chemnitz, Germany

In this talk we briefly review some basic PDE models that are used to model phase separation in materials science. They have since become important tools in image processing and over the last years semisupervised learning strategies could be implemented with these PDEs at the core. The main ingredient is the graph Laplacian that stems from a graph representation of the data. This matrix is large and typically dense. We illustrate some of its crucial features and show how to efficiently work with the graph Laplacian. In particular, we need some of its eigenvectors and for this the Lanczos process needs to be implemented efficiently. Here, we suggest the use of the NFFT method for evaluating the matrix vector products without even fully constructing the matrix. We illustrate the performance on several examples.

## Large Scale Bayesian Inversion: Sparse-Grid and Transport Methods

#### Jakob Zech<sup>1</sup>

<sup>1</sup>Heidelberg University, Heidelberg, Germany

Bayesian inference provides a rigorous framework to learn unknown parameters from noisy data. As such it has become a ubiquitous tool in modern machine learning and applied mathematics. From a practical perspective, it requires the approximation of high-dimensional integrals. Common approaches to do so include (i) sampling methods and (ii) specifically devised deterministic quadrature rules. In this talk I will discuss transport based sampling and sparse-grid quadrature as two instances of such algorithms. Additional to explaining how they can be used to perform parameter inference, I will give some insight into why these methods can deal with high dimensionality for certain PDE driven inverse problems.

## How to Prepare a Scientific Presentation

#### Friedemann Kemm<sup>1</sup>

<sup>1</sup>Brandenburg University of Technology, Cottbus-Senftenberg, Germany

This will be a talk about talks. Giving a presentation in mathematics or science in general shares some issues with presenting science in written form and with giving a public speech in general. Especially the latter is often forgotten when it comes to presenting on a conference in science and mathematics. We will discuss some general strategies how to convey a message in a public talk and some techniques in preparing slides for a presentation that will be helpful in this context.

# **Real-time Interactive Volumetric Water Simulation**

### Petr Minařík<sup>1</sup>, Ondřej Hroch<sup>1</sup>

<sup>1</sup>Keen Software House, Prague, Czechia

Water, water everywhere. This talk will focus on illustrating a modern approach to water simulation in video games. This novel solution provides realistic simulation, during real time play, and as a mechanic of play. We will share working examples of our solution, see 1, including an algorithm that illustrates our core concepts.



Figure 1: Lakes and waterfalls.

# On the Marmoreally Beautiful and yet Unlovely Sides of Einstein Equations

### Oldřich Semerák<sup>1</sup>

<sup>1</sup>Charles University, Prague, Czechia

On January 24, 1938, Albert Einstein wrote to Cornelius Lanczos: "I began with a skeptical empiricism more or less like that of Mach. But the problem of gravitation converted me into a believing rationalist, that is, into someone who searches for the only reliable source of Truth in mathematical simplicity." Mathematics (Riemann-type geometry) played a fundamental role in Einstein's quest for the new theory of gravitation. We will shortly recall when and how geometry entered the game, why the left-hand ("geometrical") side of the Einstein equations is so elegant, and why their right-hand ("physical") side likely has to remain unlovely.

# **Abstracts of Contributed Talks**

#### **Diagonally Addressed Matrices**

#### Jonas Schulze<sup>\*1</sup>, Jens Saak

<sup>1</sup>Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany

The use of mixed-precision arithmetic showed some sizeable performance improvements for dense linear algebra. Meanwhile, for sparse linear algebra due to memory bandwidth limitations the improvements are much smaller [1]. We suggest a storage scheme for sparse matrices that reduces the memory footprint at no loss of information. This has direct benefits for both classical (one-precision) and mixed-precision algorithms.

Many problems arizing in e.g. 2D and 3D FEM simulations only show local coupling between the solution components, i.e. the corresponding matrices have a very low bandwidth  $w \in \mathbb{N}$  (distance from the diagonal) under certain permutations. This suggests to store the indices of the entries in a sparse matrix not absolute but relative to the matrix diagonal, which allows to use a much smaller (integer) data type with a range of [-w, w] instead of [0, n], where  $n \in \mathbb{N}$  denotes the matrix dimensions,  $w \ll n$ . Figure 2 shows how to apply this technique to the Compressed Sparse Row (CSR) format, leading to the Diagonally Addressed CSR (DA-CSR) format.



Figure 2: Storage layout of CSR and DA-CSR. Dots represent values, bars represent the (inner) indices.

We investigate the benefits of this new storage scheme by means of the matrix vector product (SpMV), which is a basic building block of many iterative algorithms. Based on the positive definite subset of the Suite Sparse Matrix Collection [2], we observe a (up to) more than 15% performance uplift of DA-CSR having 16 bit inner indices compared to CSR having all 32 bit indices using IEEE double precision values. Current work focuses on measuring the uplift of DA-CSR for SpMV going from double to single precision values.

- [1] Abdelfattah et al, A survey of numerical linear algebra methods utilizing mixed-precision arithmetic, *The International Journal of High Performance Computing Applications* **35(4)**, 344 (2021), 10.1177/10943420211003313
- [2] Timothy A. Davis and Yifan Hu, 2011. The University of Florida Sparse Matrix Collection. *ACM Transactions on Mathematical Software* **38(1)**, Article 1 (2011), 25 pages, 10.1145/2049662.2049663

### Stopping Criteria for Coarsest-grid Solver in Multigrid V-cycle Method

#### Petr Vacek<sup>\*1</sup>, Erin C. Carson<sup>1</sup>, Kirk M. Soodhalter<sup>2</sup>

<sup>1</sup>Charles University, Prague, Czechia <sup>2</sup>Trinity College, Dublin, Ireland

Multigrid methods are frequently used when solving systems of linear equations, applied either as standalone solvers or as preconditioners for iterative methods. Within each cycle, the approximation is computed using smoothing on fine levels and solving on the coarsest level.

With growth of the size of the problems that are being solved, the size of the problems on the coarsest grid is also growing and their solution can become a computational bottleneck. In practice the problems on the coarsest-grid are often solved approximately, for example by Krylov subspace methods or direct methods based on low rank approximation; see, e.g., [1; 2]. The accuracy of the coarsest-grid solver is typically determined experimentally in order to balance the cost of the solves and the total number of multigrid cycles required for convergence.

In this talk, we present an approach to analyzing the effect of approximate coarsest-grid solves in the multigrid V-cycle method for symmetric positive definite problems. We discuss several stopping criteria derived based on the analysis and suggest a strategy for utilizing them in practice. The results are illustrated through numerical experiments.

- [1] M. Huber, Massively parallel and fault-tolerant multigrid solvers on peta-scale systems, PhD thesis, Technical University of Munich, Germany, (2019).
- [2] A. Buttari et al., Block Low Rank Single Precision Coarse Grid Solvers for Extreme Scale Multigrid Methods, *Numerical Linear Algebra with Applications*, **29**, e2407 (2022).

## A Data-driven Krylov Model Order Reduction for Large-Scale Dynamical Systems

#### Mohamed A. Hamadi<sup>\*,1,2</sup>,Khalide Jbilou<sup>1,2</sup>, Ahmed Ratnani<sup>2</sup>

<sup>1</sup>Université du Littoral Côte d'Opale, Calais, France <sup>2</sup>Université Mohammed VI polytechnique, Ben Guerir, Morocco

Some of the challenges encountered in learning dynamical systems, is the non-linearity of dynamics or in some cases, the lack of the governing physics or system's equations of motions. However, some numerically or experimentally measured data can be found. Based on this data, and using a data-driven method such as the Loewner framework, it is possible to manage this data to derive a high fidelity reduced dynamical system that mimics the behavior of the original data. In this paper, we tackle the issue of large amount of data presented by samples of transfer functions in a frequency-domain. The main step in this framework consists on computing singular value decomposition (SVD) of the Loewner matrix which provides accurate reduced systems. However, the large amount of data prevents this decomposition from being computed properly. We exploit the fact that the Loewner and shifted Loewner matrices, the key tools of Loewner framework, satisfy certain large scale Sylvester matrix equations. Using an extended block Krylov subspace method, a good approximation in a factored form of the Loewner and shifted Loewner matrices can be obtained, also, a minimal computation cost of the SVD is ensured. This method facilitates the process of a large amount of data and guarantee a good quality of the inferred model at the end of the process. Accuracy and efficiency of our method are assessed in the final section.

- [1] I.V. Gosea C.P. Vassal, A.C. Antoulas, Data-driven modeling and control of large-scale dynamical systems in the Loewner framework, (2021), https://arxiv.org/abs/2108.11870.
- [2] A.J. Mayo, A.C. Antoulas, A framework for the solution of the generalized realization problem, *Linear Algebra and Its Applications* **425(2-3)**, 634–662 (2007).

# On the Integral Equations Approach for Solving Nonlinear Inverse Elastostatics Problem

#### Mariia Vlasiuk<sup>\*1</sup>

<sup>1</sup>Ivan Franko National University of Lviv, Lviv, Ukraine

Elastostatics describes the processes of linear elasticity. We assume that D is a doubly connected bounded domain in  $\mathbb{R}^2$  with the boundary  $\partial D$  consisting of two disjoint closed  $C^2$  curves  $\Gamma_1$  and  $\Gamma_2$  such that  $\Gamma_1$ is contained in the interior of  $\Gamma_2$ . The *direct elastostatics problem* is: Given a vector function f on  $\Gamma_2$ consider the Dirichlet problem for a vector function  $u \in C^2(D) \cap C(\overline{D})$  satisfying the Navier equation and the boundary conditions

$$\mu\Delta u + (\lambda + \mu) \operatorname{grad} \operatorname{div} u = 0 \quad \text{in } D, \tag{1}$$

$$u = 0 \quad \text{on } \Gamma_1, \quad u = f \quad \text{on } \Gamma_2,$$
 (2)

where  $\mu$  and  $\lambda$  ( $\mu > 0, \lambda > -\mu$ ) are given Lamé coefficients.

The *inverse problem* is: Given the Dirichlet data f on  $\Gamma_2$  with  $f \neq 0$  and the Neumann data

$$Tu = g \quad \text{on } \Gamma_2, \tag{3}$$

determine the shape of the interior boundary  $\Gamma_1$ . Here  $Tu = \lambda \operatorname{div} u \nu + 2\mu(\nu \cdot \operatorname{grad})u + \mu \operatorname{div}(Qu)Q\nu$ , where  $\nu$  is an outward unit normal vector to the boundary, and Q is a rotational matrix. As opposed to the direct boundary value problem, the inverse problem is *nonlinear* and *ill-posed*. It can be shown, that the problem has at most one solution. In order to reduce problem to the system of nonlinear integral equations, we define functional  $G(U) = \int_{\Gamma_2} (TUf - Ug) ds$ , where f and g are given data from (2) and (3) respectively,  $U \in H^1(D)$  is a vector function that satisfies the Navier equation (1).

It can be shown that inverse boundary value problem is equivalent to the system of nonlinear integral equations. A similar approach of reducing the problem to a system of nonlinear equations is also highlighted in [1], [2]. In order to linearize system, it is essential to calculate the Fréchet derivatives of the nonlinear operators and the elastic potential with respect to the some radial function r. For solving linear system, *iteration scheme* is applied. Note that the system is linear, however it still remains incorrect due to the incorrectness of the initial problem. To obtain a stable numerical solution, the Tikhonov regularization can be applied at each iteration.

We apply quadrature method to partially discretize system of integral equations. For full discretization, we collocate the obtained relations in nodes of quadrature formulas. As a result, we get an overdetermined system of linear equations. To hande this, least squares approach can be used. For the case of a discrete problem, the method of least squares gives us a regularized system of linear equations.

The method is effective for both accurate and noisy input data. The main advantage of the proposed approach is that it is enough to measure certain data on the outside, in order to accurately reproduce the internal state of the environment.

- Chapko R., Ivanyshyn O., Kanafotskyy T. On the non-linear integral equation approaches for the boundary reconstruction in double-connected planar domains // Journal of Numerical & Applied Mathematics. – 2016. – Vol. 122. – Pp. 7–20.
- [2] Ivanyshyn O., Kress R. Nonlinear integral equations for solving inverse boundary value problems for inclusions and cracks // J. Integral Equations Appl. – 2006. – Vol. 18. – Pp. 13–38.

# **Proof Complexity of CSP**

#### Azza Gaysin<sup>\*1</sup>

<sup>1</sup>Charles University, Prague, Czechia

A wide range of well-known combinatorial problems, such as SAT, 3-coloring, graph homomorphism, and many others can be formulated in terms of Constraint Satisfaction Problem (CSP): a problem of finding an assignment of values to a set of variables, such that this assignment satisfies some specified feasibility conditions. If such an assignment exists, we call the instance satisfiable and unsatisfiable otherwise. Equivalently, CSP can be defined as a homomorphism problem between relational structures over the same vocabulary. CSP dichotomy theorem says that this class contains either NP-complete or P-time problems [1; 2].

For unsatisfiable instances of CSP, the statement that there is no homomorphism from an input structure to a target one can be expressed as a propositional tautology. Tautology is the central concept of propositional proof complexity, an area in mathematics connecting computational complexity theory and mathematical logic. The main question of proof complexity is whether NP (corresponding to SAT) is equal to coNP (corresponding to TAUT). If there is no polynomially bounded propositional proof system, then the answer is negative. The negative answer would imply that P is not equal to NP. Besides this primary goal, proof complexity embraces many other applied problems (such as SAT solvers, optimization algorithms, etc.).

We investigate the proof complexity of polynomial time CSPs. We try to prove the soundness of Zhuk's algorithm [1] solving polynomial time CSP in the theory of bounded arithmetic (with induction axiom scheme restricted to the class of bounded formulas), namely in theory  $V^1$ . If we succeed, it will imply that a class of natural tautologies is not hard for Extended Resolution proof system.

- [1] Dmitriy Zhuk, A proof of the CSP dichotomy conjecture, J. ACM, 67(5), 1-78, August 2020.
- [2] A. A. Bulatov, dichotomy theorem for nonuniform CSPs, *In 2017 IEEE 58th Annual Symposium on Foundations of Computer Science (FOCS)*, **319–330**, 2017.

# **Transport Based Sampling Using Polynomial Density Surrogates**

### Josephine Westermann<sup>\*1</sup>, Jakob Zech<sup>1</sup>

<sup>1</sup>University of Heidelberg, Heidelberg, Germany

Generating samples from arbitrary probability distributions is an integral task in various areas of modern applied mathematics such as parameter inference and uncertainty quantification. In this talk, we describe a sampling algorithm based on the Knothe-Rosenblatt transport, that can be used to approximately sample from target distributions on the *d*-dimensional unit cube  $[0, 1]^d$  under mild assumptions on the target density. The method is based on the use of polynomial density surrogates, which allows to explicitly and analytically construct the corresponding transport. We discuss efficient implementation schemes and derive error convergence rates for target densities belonging to different smoothness classes.

# **Micromechanical Parameter Identification Using Bayes Method**

#### Liya Gaynutdinova<sup>\*1</sup>, Ondřej Rokoš<sup>2</sup>, Ivana Pultarová<sup>1</sup>, Jan Havelka<sup>1</sup>, Jan Zeman<sup>1</sup>

<sup>1</sup>Czech Technical University, Prague, Czechia <sup>2</sup>Eindhoven University of Technology, Eindhoven, The Netherlands

Micromechanical parameters are required for constitutive laws that help to predict complex physical processes in materials, such as strain localisation, plasticity, delamination, and cracks. The considered problem is identification of micromechanical parameters in a fibre-reinforced plane strain composite, where the Digital Image Correlation (DIC) method is used as a full-field measurement technique [3; 4]. The existing deterministic approach to parameter identification in the form of Integrated DIC (IDIC) was shown to be overly sensitive to boundary data errors[2]. In this work, the stochastic approach is proposed, which employs a Markov Chain Monte Carlo sampling method, i.e. the Metropolis-Hastings algorithm (MHA). The identified parameters fall into two distinct groups: material and boundary condition parameters. First, the MHA that only identifies the material parameters with fixed boundary conditions is considered, and its sensitivity with respect to random and systematic errors in the boundary conditions is quantified and compared to the IDIC. MHA's parameter field is then expanded with two different ways of approximating the boundary conditions, and the method is compared to the Boundary Enriched IDIC [1]. All methods are tested with a virtual experiment that employs a Neo-Hookean hyperelastic micromechanical model, discretised with the finite element method. The benefits and pitfalls of all studied algorithms are discussed.

- [1] O. Rokoš and J.P.M. Hoefnagels and R.H.J. Peerlings and M.G.D. Geers, On micromechanical parameter identification with integrated DIC and the role of accuracy in kinematic boundary conditions, *International Journal of Solids and Structures* **146**, 241-259 (2018), 10.1016/j.ijsolstr.2018.04.004
- [2] A.P. Ruybalid and J.P.M. Hoefnagels and O. van der Sluis and M.G.D. Geers, Image-based interface characterization with a restricted microscopic field of view, *International Journal of Solids and Structures* 132-133, 218-231 (2017), 10.1016/j.ijsolstr.2017.08.020
- [3] S. Avril and M. Bonnet and A.S. Caro-Bretelle and M. Grédiac and F. Hild and P. Ienny and F. Latourte and D. Lemosse and S. Pagano and E. Pagnacco and F. Pierron, Overview of Identification Methods of Mechanical Parameters Based on Full-field Measurements, *Experimental Mechanicss* 48, 381 (2008), 10.1007/s11340-008-9148-y
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# **Euler Elastic Model & Its Treatment**

#### Sami Siddiqui<sup>\*1</sup>

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Elastic body deformation at different loading conditions has various applications in engineering, soft structures, and soft robotics. It also serves medical purposes, such as comprehending the forms of vesicles and cell membranes and their deformation behavior.

In this talk, we will consider the Euler elastica ring model subjected to uniform external force p under various loading conditions [1].



Figure 3: Elastic ring deforms under two sided symmetric uniform external force p

We will see the solution of the curvature-based nonlinear differential equation (Euler elastic equation) through different approximation methods [2] and compare the results with the Elliptic solution [3]. Furthermore, we will also discuss the stability diagram of the elastic ring using harmonic balance formulation.

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- [3] Djondjorov, P. A.; Vassilev, V. M.; & Mladenov, I. M. "Analytic description and explicit parametrisation of the equilibrium shapes of elastic rings and tubes under uniform hydrostatic pressure." *International Journal of Mechanical Sciences*, Vol. 53.5, 355-364 (2011).

# Positive Preserving Finite Volume Scheme for Surface/Subsurface Flow Simulation

### Panasun Manorost<sup>\*1</sup>, Peter Bastian<sup>1</sup>

<sup>1</sup>Heidelberg University, Heidelberg, Germany

In this work, we propose the circumcenter based finite volume method for the surface/subsurface flow model describing the dynamics of groundwater flow and overland flow model together with their interactions. The brief explanation of the model will be presented. We can prove that the scheme always provides a non-negative solution. Moreover, the numerical algorithm can be applied to the unstructured mesh. For the numerical result, we will show the simulation on a non-uniform unstructured grid with adaptive mesh refinement. Eventually, we will indicate the future work. The numerical implementation is powered by Distributed and Unified Numerics Environment (DUNE), the software framework for high-performance PDE solver.

# Time-Varying Semidefinite Programming: Path Following a Burer–Monteiro Factorization

#### Antonio Bellon<sup>\*1</sup>, Mareike Dressler<sup>2</sup>, Vyacheslav Kungurtsev<sup>1</sup>, Jakub Mareček<sup>1</sup>, André Uschmajew<sup>3</sup>

<sup>1</sup>Czech Technical University, Prague, Czechia <sup>2</sup>University of New South Wales, Sydney, Australia <sup>3</sup> University of Augsburg, Augsburg, Germany

We present an online algorithm for time-varying semidefinite programs (TV-SDPs), based on the tracking of the solution trajectory of a low-rank matrix factorization, also known as the Burer–Monteiro factorization, in a path-following procedure. There, a predictor-corrector algorithm solves a sequence of linearized systems. This requires the introduction of a horizontal space constraint to ensure the local injectivity of the low-rank factorization. The method produces a sequence of approximate solutions for the original TV-SDP problem, for which we show that they stay close to the optimal solution path if properly initialized. Numerical experiments for a time-varying Max-Cut SDP relaxation demonstrate the computational advantages of the proposed method for tracking TV-SDPs in terms of runtime compared to off-the-shelf interior point methods.

- [1] A. Bellon et al., Time-Varying Semidefinite Programming: Path Following a Burer–Monteiro Factorization, https://arxiv.org/abs/2210.08387 (2022).
- [2] A. Bellon et al., Time-Varying Semidefinite Programming: Geometry of the Trajectory of Solutions, https://arxiv.org/abs/2104.05445 (2021).

#### LECTURES ON THURSDAY AND SATURDAY WILL BE HELD AT CAMPUS KARLIN (ROOM K1, SECOND FLOOR), ON FRIDAY, AT MALA STRANA CAMPUS (ROOM S3, THIRD FLOOR)

Malá Strana Campus: Malostranské nám. 2/25, 118 00 Praha 1.

Campus Karlín: Sokolovská 49/83, 186 00 Praha 8-Karlín.

Student Meet-up on Wednesday at 7 p.m.: Campus Karlín MFF UK, Faculty of Mathematics and Physics, Room K1 (second floor), Sokolovská 49/83, 186 00 Praha 8-Karlín.

Registration: Campus Karlín MFF UK, Faculty of Mathematics and Physics, Room K1 (second floor), Sokolovská 49/83, 186 00 Praha 8-Karlín.

Conference Dinner on Thursday at 7 p.m.: Tankovna Karlín, Sokolovská 100, 186 00 Praha 8-Karlín. (Right down the street from the conference venue.)



Figure 4: Route from Campus Karlin to Tankovna Karlin.



Figure 5: Route from Florenc to Malostranské náměstí by bus no. 194.

# **Participant List**

CRISTÓBAL BERTOGLIO University of Groningen

MATOUŠ BRUNÁT Czech Technical University

> JANA BRUNÁTOVÁ Charles University

JAKUB CACH Charles University

KAUSTAV DAS Luxembourg Institute of Science and Technology | CTU Prague

> MELINA FREITAG University of Potsdam

GUILLERMO GAMBOA Charles University

PENG GAO University of Heidelberg

LIYA GAYNUTDINOVA Czech Technical University

> AZZA GAYSIN Charles University

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MAX KAHL University of Heidelberg

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> NOAMAN KHAN Charles University

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